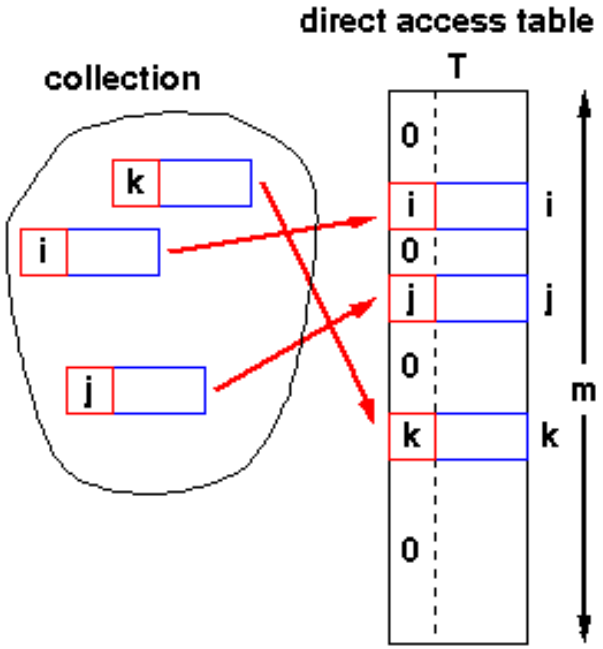
Hash Tables - Structure

- **Simplest case:**
  - Assume items have integer keys in the range $1 .. m$
  - Use the value of the key itself
    to select a slot in a
    direct access table
    in which to store the item
  - To search for an item with key, k,
    just look in slot k
    - If there's an item there,
      you've found it
    - If the tag is 0, it's missing.
  - Constant time, $O(1)$



Hash Tables - Constraints

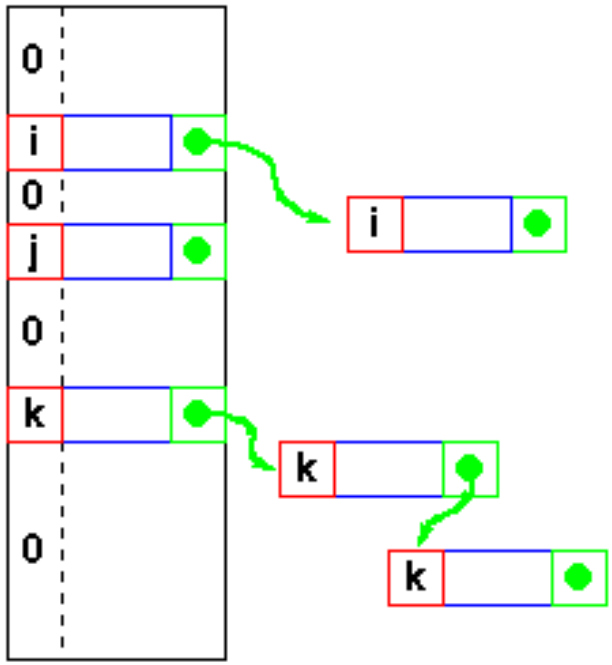
- **Constraints**
  - **Keys must be unique**
  - **Keys must lie in a small range**
  - **For storage efficiency,**
    keys must be dense in the range
  - **If they're sparse (lots of gaps between values),**
    a lot of space is used to obtain speed
    - **Space for speed trade-off**

Hash Tables - Relaxing the constraints

- **Keys must be unique**
  - **Construct a linked list of duplicates "attached" to each slot**
  - **If a search can be satisfied by *any* item with key, k, performance is still $O(1)$**

    but
  - **If the item has some other distinguishing feature which must be matched, we get $O(n^{max})$**

    where n^{max} is the largest number of duplicates - or length of the longest chain

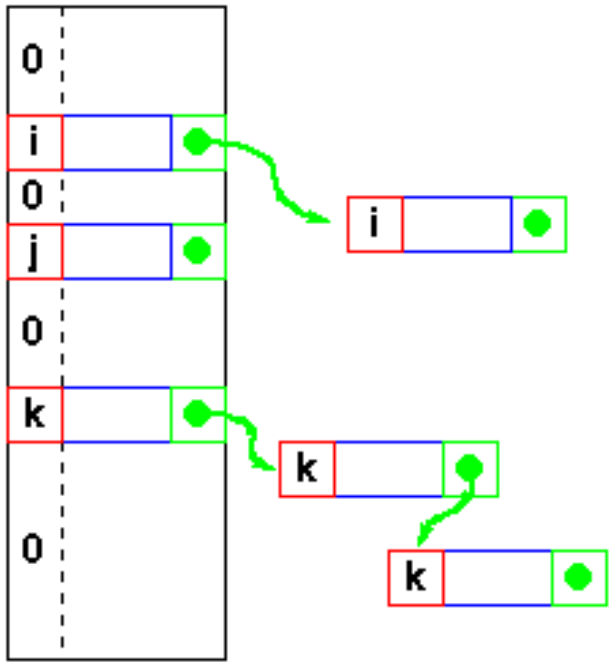


Hash Tables - Relaxing the constraints

- **Keys are integers**
  - Need a hash function

    $h(\ key\ ) \rightarrow integer$

    ie one that maps a key to an integer
  - Applying this function to the key produces an address
  - If h maps each key to a *unique integer* in the range $0 .. m\text{-}1$ then search is $O(1)$



Hash Tables - Hash functions

- **Form of the hash function**
  - Example - using an n-character key

```
int hash( char *s, int n ) {
    int sum = 0;
    while( n-- ) sum = sum + *s++;
    return sum % 256;
    }
```

    returns a value in $0 .. 255$
  - `xor` function is also commonly used

```
    sum = sum ^ *s++;
```

  - But any function that generates integers in $0..m-1$ for some suitable (*not too large*) m will do
  - As long as the hash function itself is $O(1)$!

Hash Tables - Collisions

- **Hash function**
  - **With this hash function**

```
int hash( char *s, int n ) {
    int sum = 0;
    while( n-- ) sum = sum + *s++;
    return sum % 256;
    }
```

  - **`hash( “AB”, 2 )` and `hash( “BA”, 2 )` return the same value!**
  - **This is called a collision**
  - **A variety of techniques are used for resolving collisions**

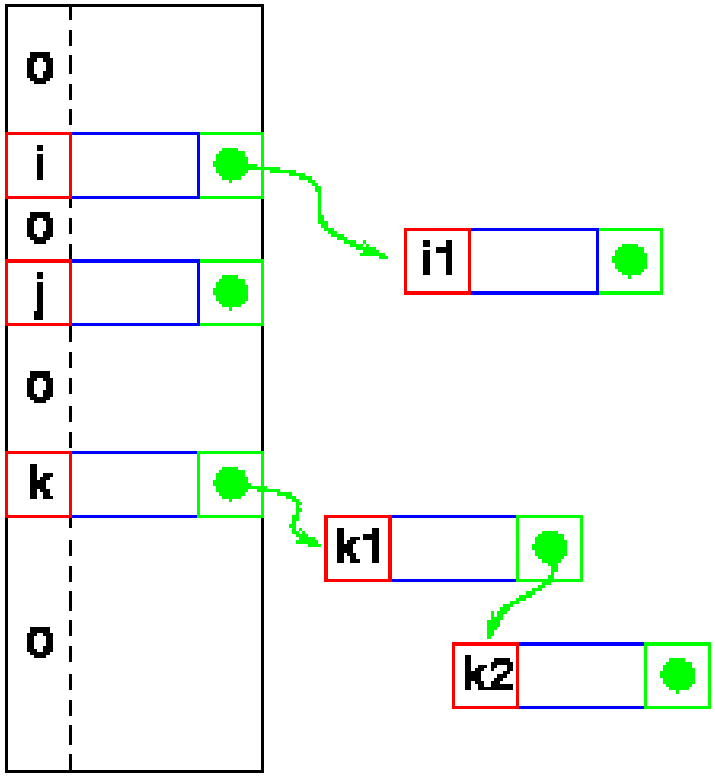
Hash Tables - Collision handling

- **Collisions**
  - **Occur when the hash function maps two different keys to the same address**
  - **The table must be able to recognise and resolve this**
  - **Recognise**
    - **Store the actual key with the item in the hash table**
    - **Compute the address**
      - **k = h(key)**
    - **Check for a hit**
      - ***if (*** **table[k].key == key** ***) then hit***
        else try next entry
  - **Resolution**
    - **Variety of techniques**

We'll look at various "*try next entry*" schemes

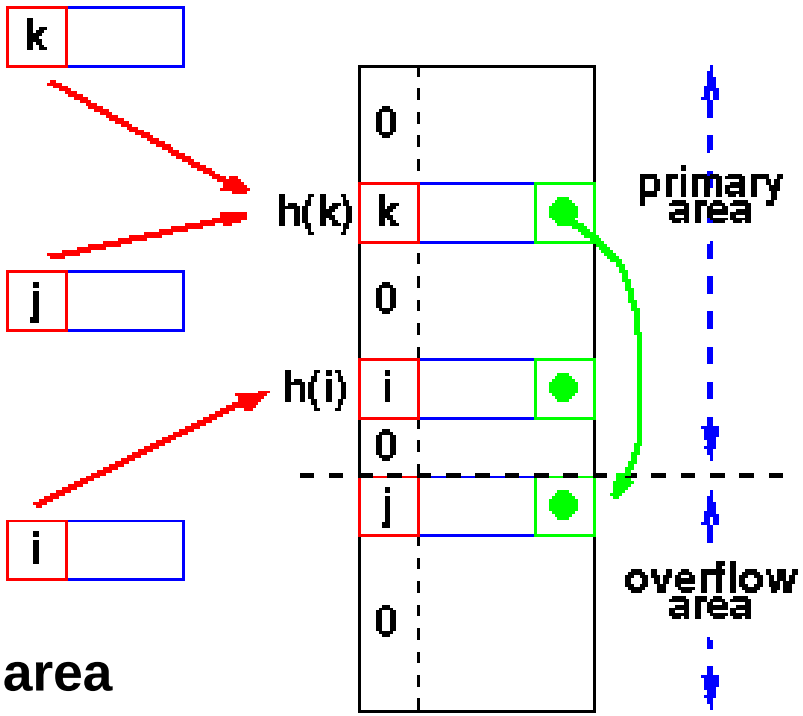
Hash Tables - Linked lists

- **Collisions - Resolution**
  - **Linked list attached to each primary table slot**
    - **h(i) == h(i1)**
    - **h(k) == h(k1) == h(k2)**
  - **Searching for i1**
    - **Calculate h(i1)**
    - **Item in table, i, doesn't match**
    - **Follow linked list to i1**
  - **If `NULL` found, key isn't in table**



Hash Tables - Overflow area

- Overflow area
  - Linked list constructed in special area of table called overflow area
- h(k) == h(j)
- k stored first
- Adding j
  - Calculate h(j)
  - Find k
  - Get first slot in overflow area
  - Put j in it
  - k's pointer points to this slot
- Searching - same as linked list



Hash Tables - Choosing the Hash Function

- **“Almost any function will do”**
  - **But some functions are definitely better than others!**
- **Key criterion**
  - **Minimum number of collisions**
    - **Keeps chains short**
    - **Maintains $O(1)$ average**

Hash Tables - Choosing the Hash Function

- **Uniform hashing**
  - **Ideal hash function**
    - $P(k)$ = probability that a key, k, occurs
    - If there are m slots in our hash table,
    - a uniform hashing function, $h(k)$, would ensure:

$$\sum_{k \mid h(k) = 0} P(k) = \sum_{k \mid h(k) = 1} P(k) = \ldots = \sum_{k \mid h(k) = m-1} P(k) = \frac{1}{m}$$

Read as sum over all k such that $h(k) = 0$

    - *or, in plain English,*
    - the number of keys that map to each slot is equal

Hash Tables - A Uniform Hash Function

- ***If*** **the keys are integers randomly distributed in** $[\,0\,,\,r\,)$**,** **Read as** $0 \le k < r$

  then

$$h(k) = \left\lfloor \frac{mk}{r} \right\rfloor$$

  is a uniform hash function

- **Most hashing functions can be made to map the keys to** $[\,0\,,\,r\,)$ **for some** r
  - ***eg*** **adding the ASCII codes for characters mod 255 will give values in [0, 256) or [0, 255]**
  - **Replace + by xor**

    ⇐same range without the mod operation

Hash Tables - Reducing the range to $[\,0, m\,)$

- **We've mapped the keys to a range of integers**
  $$0 \le k < r$$
- **Now we must reduce this range to** $[\,0, m\,)$
  where m **is a reasonable size for the hash table**

- **Strategies**
  - **Division - use a mod function**
  - **Multiplication**
  - **Universal hashing**

Hash Tables - Reducing the range to [0, *m*)

✶ Division

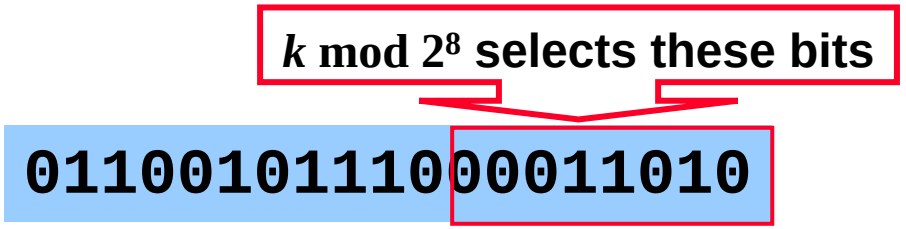
- Use a mod function

$$h(k) = k \bmod m$$

- Choice of m?
  - Powers of 2 are generally not good!

    $h(k) = k \bmod 2^n$

    selects last n bits of k



    - All combinations are not generally equally likely
- Prime numbers close to 2^n seem to be good choices

  eg want ~4000 entry table, choose $m = 4093$

Hash Tables - Reducing the range to $[\,0, m\,)$

✸ Multiplication method

- Multiply the key by constant, A, $0 < A < 1$
- Extract the fractional part of the product

  $(\,kA - \lfloor kA \rfloor\,)$
- Multiply this by m

  $h(k) = \lfloor m * (\,kA - \lfloor kA \rfloor\,) \rfloor$
- Now m is not critical and a power of 2 can be chosen
- So this procedure is fast on a typical digital computer
  - Set $m = 2^p$
  - Multiply k (w bits) by $\lfloor A \cdot 2^w \rfloor$ ← $2w$ bit product
  - Extract p most significant bits of lower half
  - $A = ½(\sqrt{5} - 1)$ seems to be a good choice (*see* Knuth)

Collision Frequency

- **Birthdays *or* the von Mises paradox**
  - **There are 365 days in a normal year**
    - **←Birthdays on the same day unlikely?**
  - **How many people do I need before "it's an even bet" (*ie* the probability is > 50%) that two have the same birthday?**
  - **View**
    - **the days of the year as the slots in a hash table**
    - **the "birthday function" as mapping people to slots**
  - **Answering von Mises' question answers the question about the probability of collisions in a hash table**

Distinct Birthdays

- **Let $Q(n)$ = probability that n people have distinct birthdays**
- $Q(1) = 1$
- **With two people, the 2nd has only 364 "free" birthdays**

$$Q(2) = Q(1) * \frac{364}{365}$$

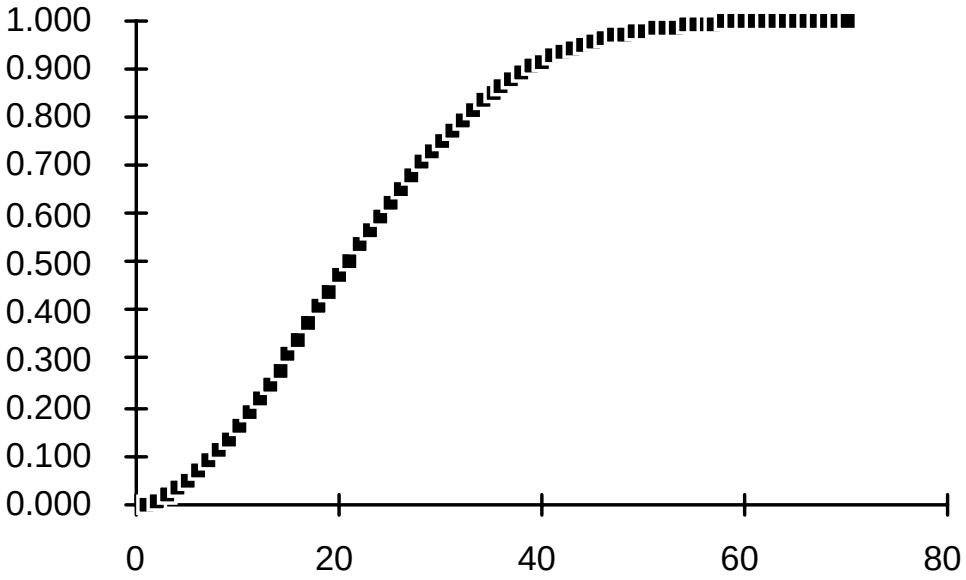
- **The 3rd has only 363, and so on:**

$$Q(n) = Q(1) * \frac{364}{365} * \frac{364}{365} * \ldots * \frac{365-n+1}{365}$$

Coincident Birthdays

- **Probability of having two identical birthdays**
- $P(n) = 1 - Q(n)$
- $P(23) = 0.507$

- **With 23 entries, table is only 23/365 = 6.3% full!**



Hash Tables - Load factor

- **Collisions are very probable!**
- **Table load factor**

$$\alpha = \frac{n}{m}$$

n = number of items
m = number of slots

  must be kept low
- **Detailed analyses of the average chain length (or number of comparisons/search) are available**
- **Separate chaining**
  - **linked lists attached to each slot**

gives best performance

  - **but uses more space!**

Hash Tables - General Design

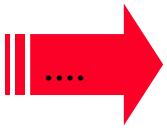
☆ **Choose the table size**

- **Large tables reduce the probability of collisions!**
- **Table size, m**
- **n items**
- **Collision probability $\alpha = n / m$**

🕐 **Choose a table organisation**

- **Does the collection keep growing?**
  - **Linked lists (..... but consider a tree!)**
- **Size relatively static?**
  - **Overflow area *or***
  - **Re-hash**

🕐 **Choose a hash function**



Hash Tables - General Design

- **Choose a hash function**
  - **A simple (and fast) one may well be fine ...**
  - **Read your text for some ideas!**
- **Check the hash function against your data**
  - **Fixed data**
    - **Try various h, m until the maximum collision chain is acceptable**

    ←Known performance
  - **Changing data**
    - **Choose some representative data**
    - **Try various h, m until collision chain is OK**

    ←Usually predictable performance