University of Veszprém
Complexity Theory Assignment
26 November 2002

Duration: 2 hours

Name of student:

1. Describe and explain the steps of *NP*–completeness proof. [4 marks]
2. Prove that if a set is recursive, then it is recursively enumerable. [3 marks]
3. Prove that class *P* is closed under complement. [3 marks]
4. Define co*NP* to be the class of languages defined as $\{\overline{L} : L \in NP\}$. That is, co*NP* is the set of languages whose complement is in *NP*. Prove that if *P* = *NP* then *NP* = co*NP*. You are given that *P* is closed under complement. [2 marks]
5. For each of the following statements, determine whether it is correct / incorrect, yes / no, or not sure. Explanation is needed to support your answer.
   a) Problems in *NP* cannot be solved in polynomial time. [2 marks]
   b) Problems in *NP* can be solved in exponential time. [3 marks]
   c) Given two problems X and Y, does X ≤ Y imply Y ≤ X? [2 marks]
   d) We know that both SAT and Clique are *NP*–complete. Is SAT ≤ Clique? Is Clique ≤ SAT? [3 marks]
6. Explain the difference between a *deterministic* and a *non–deterministic* Turing machine. [2 marks]
7. A University has *n* clubs and societies, the largest of which contains *m* members (of course, students can be members of multiple clubs and societies). The Rector of the University wishes to hold a dinner in honour of such student activities. Unfortunately, the Hall can seat comfortably only *k* guests. The Rector's problem is as follows: can he construct a guest list of *k* students such that every club and society has at least one member in attendance? You must prove that this problem is *NP*–complete. You are given only that the problem SATISFIABILITY is *NP*–complete. [3 marks]
8. Consider the *HALTING* problem of determining whether a given function *p*() terminates or not. Consider also the problem *CHANGE* to determine whether a given variable *a* changes value during the execution of function *q*(). Prove that *HALTING* ≤ *CHANGE*. [3 marks]
9.
   a) Give the definition of *phase transition*. [1 mark]
   b) Give the definition of *crossover point*. [1 mark]
   c) Explain the construction of the phase transition plot. Give an example. [2 marks]
   d) Prove that $K = -\frac{\sum_{i=0}^{n-1} \log \frac{(e-i)}{(n(n-1)/2)-i}}{\log((n-1)!/2)}$. [3 marks]

Optional:

10. Which is larger (for large *n*), $n^{\log n}$ or $(\log n)^n$? [5 marks]